

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

Bilingual Mathematics – Arguments pro

Bilingualism was developed to enhance intercultural competence. David Marsh's definition of CLIL amplified the thinking about bilingual education. Mathematicians have been reluctant to adopt their domain to bilingual teaching and not many effort were made. (Rolka (6)) This text gives a new approach to bilingual mathematics. The approach is based on a great spectrum of arguments which may be applied to different mathematical topics. Most successful so far were efforts teaching geometry or using constructive materials from a *Mathematical Laboratory*. This means just a bunch of material being useful for pupil centred classroom work which has shown to be preferable in bilingual teaching.

The argumentation is an argumentation *pro Bilingual Math* and bundles its arguments under

- I) Content arguments
- II) Language arguments derived from the "language of math"
- III) Motivational arguments for the use of geometry in the bilingual classroom
- IV) Cognitive arguments

Mathematics is said to be difficult. Young academics, excellent English teachers refuse the idea to cover mathematical ideas in their English lessons: "I haven't even checked it at school!". But of course this can't be the truth for all of them. Many of them must have performed well in mathematics.

So one has to ask, what do those young teachers really mean, if they refuse? Perhaps: "Math is not important enough" or "The stories of the pirates of the Carabbean are more amusing for young children than LEGO robots". May be there deeper arguments like: "How should I teach children with such poor language knowledge in mathematical terminology an contents.

The actual solution, e.g. published recently by Wildhage/Otten, *Praxis des bilingualen Unterrichts* (1), is that of the language-content-immersion. I practised in my project of bilingual mathematical this instructional technology.

After carefully and reflected choice of mathematical themes the lessons were performed in a way, that separates language and content layers. There should be never or only short term interference of difficulties of the two types.

The question remains, is all maths is suitable to be taught in bilingual math education? As rather young children (age 8 to 12) are, we may use thoughts of Maria Montessori (4) as well as from Heinz von Förster (2) and Hans Freudenthal (3).

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

These very different people, outstanding in their contribution to education and mathematics, referred to a certain use of constructional material in the teaching process.

„Wenn man den Kindern sagt, jetzt müsst ihr eine Mathematik erfinden, dann sind sie ganz begeistert. (2, S. 132)“ and

„Zuerst erfindet man, und dann entdeckt man, was die Erfindungen alles mit sich bringen. (2, S. 134) „ are two very inspiring ideas of Heinz von Förster, referring to the children action constructing mathematics out of the given tasks.

„Ich möchte, dass der Schüler nicht angewandte Mathematik lernt, sondern lernt, wie man Mathematik anwendet.“ (Freudenthal in 3), with this idea Freudenthal rejects a very common misconception of applied mathematics in school.

As I had the pleasure to discuss with Prof. Freudenthal personally, when I was a young teacher, I am totally sure that he would put great emphasis on pupil centred teaching strategies and individualised instruction.

In this article I try to use these ideas in order to construct an outline of the didactic of bilingual mathematics education.

Reasons for Bilingual Mathematic

I) Content arguments

1) Global academic world

Many efforts that have gone into multilingual or bilingual education have tried to convey a competence in every day life or are inspired of cross cultural competence. My efforts are focused on an additional quality. In scientific and engineering core knowledge

- English is the lingua franca
- Education gets more and more internationalised and
- This is true for the labour as well as for the staff.

So it is useful to give children multilingual access to their own knowledge.

2) Educating for elaborated professional tasks

Science and mathematics require a special use of language and methods, which differ much from those of the humanities. But not even basic terms of mathematics appear in the everyday English lesson. Such may be: Isosceles triangle, protractor, compass, median. Also lots of methods are necessary needing vocabulary round reporting, reasoning, documentation etc.

Especially in documentation more elaborate phrases are necessary to be able to argue, to considerate, to prove, to refuse.

In the effort to add to the pupil's scientific knowledge and performance competences in both bilingual languages have to be conveyed.

3) Adding academic competence to preferred students

Till today I have done two projects on bilingual mathematics with totally different aims. In 2002/2003 I had a bilingual course of computer science with 18-year-old students. It aimed at students who were preparing for scientific and engineering studies at international and German universities. Students of physics at the university of Augsburg e.g. in their post-bachelor studies have to attend science lectures in English and most of the publications in computer science literature are published in English.

Of course I simply could have my course named "computer science in English". But indeed we undertook great effort to keep the knowledge available in both languages, namely English and German, avoiding "Denglish"

II) Language arguments and mathematical language

4) Mathematics or language first?

Wildhagen/Otte pointed out: *"(tr.) The core problem of bilingual classes is the discrepancy between cognitive and lingual possibilities of the learners, that means the gap between what they want to express and what they can say."*

This may be true for the fields the authors are dealing with, i.e. history and geography. In my opinion things are different in a mathematical context. In

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

mathematics the number of nouns is very small and the denoted objects share very few relations. For example with "*point*" and "*line*" and the "*incidence relation*" a whole geometry can be constructed.

This restriction of the number of objects and relations is typical for mathematical thinking. It reduces the number of words the students must know but not the richness of mathematical thinking..

So what should we teach first, the language or the content? Also in Mathematics best strategy is to avoid the problem. This can be done with a pendulum strategy between both languages. And in addition, mathematical contents reduce the language problems by its special way of thinking.

5) Mathematics as a game

Very common mathematics is seen to be a finished task. For continuing my argumentation I want to rise another point of view: That of a science working in a constructed world.

Maybe that chess is a popular example. There a 2x16 chessman of 5 types, 8x8 fields and 6 rules for moves and some rules for ending the game. One can think, that not many "meaningful sentences" can be spoken within a "chess language". But think a move to be a sentence there are almost infinite possibilities to form essays, consisting from normally 3 to 50 sentences.

Thinking about math as a game one finds for the didactic of mathematics especially for younger learners a great and sometimes surprising variety of ideas for bilingual lessons.

6) Mathematics: A Language to argue and to reason

Do we really know how pupils learn mathematics? The answer is: We don't know much. Despite efforts from 1960 to 1980 there has not been much progress. An international group, the IGPME (now PME) was founded to study the psychology of mathematical education. An enormous progress could be made by Kathleen Hart in changing the methods of observation from questionnaire to interview, where the test person has to say aloud what he is his thinking. The researchers found a way

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

to the "internal dialogue". This is a key to the internal representation of knowledge and also to the misconcepts.

Research is on a good way but it will last long until we can draw consequences for a better instructional practise.

At the moment we can just notice, that maths lessons are suited to make students argue and are able to stimulate then internal flow of thoughts also in the foreign language.

But be careful: The normal maths lesson in Europe is more a teachers monologue with rare pupils' answers. That's not, how bilingual mathematic lessons should look be, other communications patterns are more useful.

III) Motivational arguments for geometry

7) Adding an attraction to math lessons for young learners

This year I taught geometry parts in my class bilingually. The reaction of the class was friendly and all in all it was a success. But not all of the pupils accepted to be taught in English. Some of them, interestingly enough the best girls, insist on answering German to my English questions. This habit vared and they asked me only sometimes to repeat something in German, probably to make sure they had understood correctly

I practised pupil-centred-lessons with a lot of possibilities to interact. The communication, at first in German, changed gradually into a rudimentary English dialog. This change can be studied in the (non professional) video of 4 lessons.

Additional computer use helped to rise interest.

8) Constructing is fun

Geometry is new for pupils in the 7th grade. This new world of definition and construction pleases pupils with suitable talents. They love this new field of mathematics. The construction of a figure by a logical sequence of actions is fascinating for them. They move in their mind through chains of possibilities, When

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

they have found a solution they transform it from mental operation into language and action.

This explains why I have chosen geometry as content for my bilingual classes: You must go through many thoughts before you speak and act.

9) Arguing is social interaction

Geometry is construction and argumentation and less schematic writing than algebra may be. When one observes groups working together to solve mathematical problems one can see, that interaction between pupils is much higher when they do geometry instead of algebra. For the bilingual classes this is there a chance to make students speak English in the classroom interaction: *"Which radius, where is the centre, lend me your compass, enlarge the line, this may not be the right point because, if this was right then..."*

As a result a natural bilingual interaction is created and the teacher could join into a differentiated classroom discourse.

IV) Cognitive arguments

10) Language L3 and meta language

The pendulum technique in bilingual classes seems to be best practice. It alters between L1=normal classroom language L2=bilingual language and L3=content specific language.

Sometimes new facts are available neither in L1 nor in L2. One has to build up the nominators in L1 and L2 by a definition, which may be the reference to an object, by examples or by descriptions. Think e.g. how to introduce the words "atom" or "line".

Elementary mathematics uses a crayon, circle and ruler to exemplify the meaning of "point", "line" and "circle".

But in the scheme of mathematical thinking, those real objects just represent the virtual objects, who really are the objects of the mathematical game.

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

Seen in that perspective, compass and ruler together with their use build up a meta language.

Each mathematics teacher knows, that there is a good wile until the young learners discriminate the real object from the defined (virtual) object. This discrimination is necessary: If in a drawing three bisecting perpendiculars do not intersect in one point this may have many reasons. That they "really" intersect in one and only one point in any triangle can only be assured by a proof. If the student recognizes the necessity of a proof he has got the difference between the object of thinking and the representation of the object and so he has done the first essential step to geometry. What can be seen in the math classroom very frequently is the pendulum technique between language and meta language for making clear "what is really meant".

11) Social interaction and maths combined leads to thigh sub verbal flow of thoughts

Bilingual mathematics should try to bring the students into verbal action. So pupil centred teaching is preferable. In that case the teacher's role changes. He is more a coach. This change can be supported by constructive materials, such as I described to be in the Mathematical Laboratory.

It is the quality o these materials that makes quick and easy answers impossible. Whenever the teacher has to or wants to interfere he must analyse the steps already taken by the pupil. Then he takes a suggesting and helping role to keep the group going.

So this classroom organization in combination with suitable materials creates a vivid discourse between the pupils and the teacher as well in the bilingual language.

There is even the possibility that the internal stream of thinking may change into the foreign language.

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

12) Multicentred brain activation in bilingual mathematics

Today we know, that geometrical and language faculties are situated in diverse regions of the brain. It is also true, that learning has the tendency to produce new neuronal contacts. Narrow training in algebra is connected to language operating and rhythm feeling and is therefore preferred by female students. Training in space orientation and geometry activates brain areas for orientation and construction and we know of certain preferences of male students.

Teacher constructing materials for highly talented pupils tend to look for tasks in which geometrical and algebraic tasks are integrated. Thus they activate simultaneously the two cerebral hemispheres.

Bilingual mathematics activates more centres of the brain than a mere lesson in the mother tongue, because mother tongue and foreign language are implemented in different brain areas (see Wattendorf et al. (5))

13) Bilingual useful learning materials shows to be also learning materials for highly talented children

The construction of materials for highly talented pupils differs significantly from those in standard textbooks. These materials can be characterised as unusual problems with overlapping tasks and constructive activities. These materials very often are presented in varying social situations.

Long term research with this type of material shows, that complex materials, supposedly by activating more parts of the brain, help the children to develop wider spans of concentration. By the way, this material with its enormous stimulus, is not more difficult. So it is ideal material for all bilingual instruction. The main difference between high potentials and average children working with these materials is simply, that the highly gifted children draw more content out of the same work and are better at transfer to another situation.

To sum up, learning materials constructed for the highly talented have many traits of the materials I found useful for bilingual mathematics.

Ein Sokrates Projekt, gefördert durch EU Mittel, www.europabilingual.net

- 1) Wildhage, Otten, Praxis des Bilingualen Unterrichts, Cornelsen Verlag
- 2) Heinz von Förster, Wie wir uns erfinden, Carl-Auer-Systeme Verlag, 2004
- 3) Freudenthal Hans, Freudenthal, Hans: Mathematik als pädagogische Aufgabe, I/II, Klett Verlag, 1970
- 4) Montessori Maria, Kinder sind anders, Klett-Cotta Verlag, 1952
- 5) Wattendorf et al, Different languages activate different subfields in Broca's are, Neuroimage, 13(6), june 2001
- 6) Rolka, Katrin, BILINGUAL LESSONS AND MATHEMATICAL WOLRD VIEWS – A GERMAN PERSPECTIVE, Universität Duisburg